A SEARCH-BASED APPROACH TO ANNEXATION AND MERGING IN WEIGHTED VOTING GAMES

Ramoni O. Lasisi and Vicki H. Allan
Department of Computer Science, Utah State University, Logan, U.S.A.
ramoni.lasisi@aggiemail.usu.edu, vicki.allan@usu.edu

Keywords: Agents, Weighted voting games, Power indices, False-name manipulation, Annexation, Merging.

Abstract: Weighted voting games are classic cooperative games which provide a compact representation for coalition formation models in multiagent systems. We consider manipulation in weighted voting games via *annexation* and *merging*, which involves an agent or some agents misrepresenting their identities in anticipation of gaining more power at the expense of other agents in a game. We show that annexation and merging in weighted voting games can be more serious than as presented in the previous work. Specifically, using similar assumptions as employed in a previous work, we show that manipulators need to do only a polynomial amount of work to find a *much improved* power gain, and then present two *search-based* pseudo-polynomial algorithms that manipulators can use. We empirically evaluate our search-based method for annexation and merging. Our method is shown to achieve significant improvement in benefits for manipulating agents in several numerical experiments. While our search-based method achieves improvement in benefits of over 300% more than those of the previous work in annexation, the improvement in benefits is 28% to 45% more than those of the previous work in merging for all the weighted voting games we considered.

1 INTRODUCTION

False-name manipulation in *weighted voting games* (WVGs), which involves an agent or some agents misrepresenting their identities in anticipation of power increase, has been identified as a problem. This is because the anticipated power gain by manipulating agents is at the expense of other agents in the game. The menace can take different forms. With *annexation*, an agent, termed, an *annexer*, takes over the voting weights of some agents in a game. Power is not shared with the annexed agents. Forming an *alliance* or *merging* involves voluntary merging of weights by two or more agents to form a single bloc (Machover and Felsenthal, 2002; Aziz et al., 2011; Lasisi and Allan, 2011). Merged agents expect to be compensated with their share of the power gained by the bloc. The agents whose voting weights are taken over or merged into a bloc are referred to as *assimilated* agents. When agents engage in these manipulations, it becomes difficult to establish or maintain trust, and more importantly it becomes difficult to assure fairness in such games.

WVGs are classic cooperative games which provide a compact representation for coalition formation models in multiagent systems. Each agent in a WVG has an associated weight. A subset of agents whose total weight meets or exceeds a specified *quota* is called a *winning coalition*. The weights of agents in a game correspond to resources or skills available to the agents, while the quota is the amount of resources or skills required for a task to be accomplished. For example, in *search and rescue*, robotic agents put their resources (i.e., weights) together in large natural disaster environments to reach the necessary levels (i.e., quota) to save life and property.

We are concerned with the ways in which agents that complete a task are compensated from their jointly derived payoff, taking into account each agent's resource (weight) contribution. The relative power of each agent reflects its significance in the elicitation of a winning coalition. Although a larger weight by an agent makes it more likely that an agent can affect the outcome of a WVG, the weight of an agent in a game is not always proportional to its power (Aziz et al., 2011). A widely accepted method for measuring such relative power in WVGs uses *power indices*. The two best-known and most used indices for measuring power in WVGs are *Shapley-Shubik* (Shapley and Shubik, 1954) and *Banzhaf* (Banzhaf, 1965) power indices.

WVGs can be viewed as a form of competition among agents to share the available *fixed* power whose total value is always assumed to be 1. Agents may thus resort to a form of false-name manipulation (annexation or merging) to improve their influence in anticipation of gaining more power. This paper continues the work studied originally by (Machover and Felsenthal, 2002), (Aziz et al., 2011), and (Lasisi and Allan, 2011) on annexation and merging in WVGs. We extend the framework of (Lasisi and Allan, 2011) on susceptibility of power indices to annexation and merging in WVGs to consider a *much improved* power gain or *benefit* for manipulators.

Consider a WVG of n agents. The simulation of (Lasisi and Allan, 2011) for annexation and merging is based on a *random* approach where some agents, say $k < n$, in the game are randomly selected to be assimilated in annexation, and to form a voluntary bloc of manipulators in merging. This simple random approach shows that on average, annexations can be effective for manipulators using both the Shapley-Shubik and Banzhaf power indices to compute agents' power. Their results also show that merging only has a minor effect on the power gained for manipulators using the Shapley-Shubik index, while it is typically non-beneficial (i.e., no power is gained) for manipulators using the Banzhaf index. We note that randomly selecting the k agents to be assimilated for both annexation and merging this way fails to consider the benefits of a more strategic approach.

We show that manipulation via annexation and merging can be more serious than as presented in the previous work. Specifically, we show, using similar assumptions for annexation and meging as employed in the simple random simulation of (Lasisi and Allan, 2011), that manipulators need to do only a *polynomial* amount of work to find a much improved power gain during manipulation. Given that the problem of computing the Shapley-Shubik and Banzhaf power indices of agents is already NP-hard, and only *pseudo-polynomial* or approximation algorithms are available to compute agents' power, we then present two *search-based* pseudo-polynomial time algorithms that manipulators can use to find a much improved power gain. Furthermore, for reasons of efficiency, we do not implement the two algorithms exactly. Rather, we employ *informed heuristic search strategies* to complement the performance of the algorithms, while taking into consideration the two power indices in the design of the heuristics.

We empirically evaluate our search-based method for annexation and merging. Our method is shown to achieve significant improvement in benefits over previous work for manipulating agents in several numerical experiments. Thus, unlike the simple random simulation of (Lasisi and Allan, 2011) where merging has little or no benefits for manipulators using both the Shapley-Shubik and Banzhaf indices, results from our experiments suggest that manipulation via merging can be *highly* effective for manipulators. The simple random approach to manipulation via annexation and merging seems *unintelligent*, thus, it is impractical that strategic agents would be keen in employing such method. In view of this, we modify the simple random approach to select the best power gain or benefit from three random choices (which we refer to as *best-of-three*) and compare with our search-based method. We note that this simple modification provides higher average benefits to the manipulators than those of the simple random approach.

The remainder of the paper is organized as follows. Section 2 provides some preliminaries. Section 3 demonstrates examples of annexation and merging in WVGs. We present our search-based approach to annexation and merging in Section 4. In Section 5, we consider informed heuristic search strategies to complement the performance of the search-based method. In Section 6, we present results of empirical evaluation of our search-based method. Section 7 discusses related work. We conclude in Section 8.

2 PRELIMINARIES

2.1 Weighted Voting Games

Let $I = \{1, \cdots, n\}$ be a set of n agents and the corresponding positive weights of the agents be $\mathbf{w} = \{w_1, \cdots, w_n\}$. Let a coalition $S \subseteq I$ be a non-empty subset of agents. A WVG G with *quota* q involving agents I is represented as $G = [w_1, \cdots, w_n; q]$. Denote by $w(S)$, the weight of a coalition, S, derived as the summation of the weights of agents in S, i.e., $w(S) = \sum_{j \in S} w_j$. A coalition, S, wins in game G if $w(S) \geq q$, otherwise it loses. WVGs belong to the class of *simple voting games*. In simple voting games, each coalition, S, has an associated function $v : S \to \{0, 1\}$. The value 1 implies a win for S and 0 implies a loss. So, $v(S) = 1$ if $w(S) \geq q$ and 0 otherwise.

2.2 Power Indices

We provide brief descriptions of the two power indices we use in computing agents' power in WVGs. For further discussion, we refer the reader to (Felsenthal and Machover, 1998; Laruelle, 1999).

Shapley-Shubik Power Index

The Shapley-Shubik index quantifies the marginal contribution of an agent to the *grand coalition* (i.e., a coalition of all the agents). Each permutation (or ordering) of the agents is considered. We term an agent to be *pivotal* in a permutation if the agents preceding it do not form a winning coalition, but by including this agent, a winning coalition is formed. Shapley-Shubik index assigns power to each agent based on the proportion of times it is pivotal in all permutations. We specify the computation of the power index using notation of (Bachrach et al., 2010). Denote by π a permutation of the agents, so $\pi : \{1,\ldots,n\} \to \{1,\ldots,n\}$, and by Π the set of all possible permutations. Denote by $S_\pi(i)$ the predecessors of agent i in π, i.e., $S_\pi(i) = \{j : \pi(j) < \pi(i)\}$. The Shapley-Shubik index, $\varphi_i(G)$, for each agent i in a WVG G is

$$\varphi_i(G) = \frac{1}{n!}\sum_{\pi\in\Pi}[v(S_\pi(i)\cup\{i\}) - v(S_\pi(i))]. \qquad (1)$$

Banzhaf Power Index

The Banzhaf power index bases power on an agent being able to turn a losing coalition into a winning coalition by its vote. An agent $i \in S$ is referred to as being *critical* in a winning coalition, S, if $w(S) \geq q$ and $w(S\backslash\{i\}) < q$. The Banzhaf power index computation for an agent i is the proportion of times i is critical compared to the total number of times any agent in the game is critical. The Banzhaf index, $\beta_i(G)$, for each agent i in a WVG G is given by

$$\beta_i(G) = \frac{\eta_i(G)}{\sum_{j\in I}\eta_j(G)} \qquad (2)$$

where $\eta_i(G)$ is the number of coalitions for which agent i is critical in G.

2.3 Annexation and Merging

Let G be a WVG. Let Φ be any of Shapley-Shubik or Banzhaf power indices. We denote the power index of an agent i in G by $\Phi_i(G)$. Also, consider a coalition $S \subseteq I$, we denote by $\&S$ a bloc of assimilated voters formed by agents in S. We say that a power index Φ is *susceptible* to manipulation whenever a WVG G is *altered* by an agent i (in the case of annexation or some agents in the case of merging) and such that there exists a new game G' where $\Phi_i(G') > \Phi_i(G)$. In other words, Φ is susceptible to manipulation when the power index of the agent in the altered game is more than its power index in the original game.

Definition 1. *(Manipulation by Annexation).*

Let agent i alter game G by annexing a coalition S (i assimilates the agents in S to form a bloc $\&(S \cup \{i\})$). We say that Φ is susceptible to manipulation via annexation if there exists a new game G' such that $\Phi_{\&(S\cup\{i\})}(G') > \Phi_i(G)$; the annexation is termed *advantageous*. The *factor of increment* by which the annexer gains is given by $\frac{\Phi_{\&(S\cup\{i\})}(G')}{\Phi_i(G)}$. If $\Phi_{\&(S\cup\{i\})}(G') < \Phi_i(G)$, then the annexation is *disadvantageous*.

Definition 2. *(Manipulation by Merging).*

Let a manipulators' coalition, S, alter G by merging into a bloc $\&S$. We say that Φ is susceptible to manipulation via merging if there exists a new game G' such that $\Phi_{\&S}(G') > \sum_{j\in S}\Phi_j(G)$; the merging is termed *advantageous*. The factor of increment by which the manipulators gain is given by $\frac{\Phi_{\&S}(G')}{\sum_{j\in S}\Phi_j(G)}$. If $\Phi_{\&S}(G') < \sum_{j\in S}\Phi_j(G)$, then the merging is *disadvantageous*. The agents in a bloc formed by merging are assumed to be working cooperatively and have transferable utility. For the sake of simplicity in our analysis, we also refer to the factor of increment as power gain or benefit.

3 EXAMPLES OF ANNEXATION & MERGING IN WVGs

We provide examples to illustrate annexation and merging in WVGs. We have used Banzhaf power index as a reference for these examples. The annexer and assimilated agents are all shown in bold.

Example 1. *(Manipulation by Annexation).*

Let $G = [\mathbf{12}, 16, \mathbf{18}, \mathbf{19}, 23, 26, 43, 46, 50; 195]$ be a WVG. The power index of agent 1 with weight 12 is $\beta_1(G) = 0.026$. Suppose the agent annexes agents 3 and 4 with weights 18 and 19. An assimilated bloc of weight 49 is formed in the new game $G' = [\mathbf{49}, 16, 23, 26, 43, 46, 50; 195]$. The new power index of the annexer $\beta_1(G') = 0.177 > \beta_1(G)$. The agent gains from the annexation and increases its power index by a factor of $\frac{0.177}{0.026} = 6.81$.

Example 2. *(Manipulation by Merging).*

Let $G = [12, 16, 18, 19, 23, \mathbf{26}, \mathbf{33}, \mathbf{40}, \mathbf{45}; 155]$ be a WVG. The last four agents in the game are designated as would-be manipulators. The Banzhaf power indices of these agents are: $\beta_6(G) = 0.116$, $\beta_7(G) = 0.142$, $\beta_8(G) = 0.174$, and $\beta_9(G) = 0.200$. So, $\sum_{j=6}^{9}\beta_j(G) = 0.632$. Suppose the agents decide to merge their weights. A merged bloc of weight 144 is formed in the new game $G' = [12, 16, 18, 19, 23, \mathbf{144}; 155]$. The power index of the bloc $\beta_6(G') = 0.861 > 0.632$. The manipulators gain from the merging and increase their power indices by a factor of $\frac{0.861}{0.632} = 1.36$.

There exist examples where the two forms of manipulation may not be beneficial using the two power indices. However, (Machover and Felsenthal, 2002) have shown that, in the case of annexation, it is *always* beneficial for an annexer to assimilate other agents using Shapley-Shubik power index.

4 SEARCH-BASED APPROACH TO ANNEXATION & MERGING

4.1 Overview

As noted in the introduction, randomly selecting agents to be assimilated in annexation, or to form a voluntary bloc in merging fails to consider the benefits of a more strategic approach to false-name manipulation. In this section, we extend the simple random simulation of (Lasisi and Allan, 2011) for annexation and merging in WVGs. We propose a search-based approach for the two forms of manipulation (annexation and merging) using the Shapley-Shubik and Banzhaf indices to compute agents' power.

In considering our search-based approach to annexation and merging, we have implemented two pseudo-polynomial manipulation algorithms, one for each form of manipulation. To begin with, we recall that the problem of calculating the Shapley-Shubik indices and Banzhaf indices for WVGs is NP-hard, and both admit pseudo-polynomial algorithms using dynamic programming (Matsui and Matsui, 2000; Matsui and Matsui, 2001) or generating functions (Brams and Affuso, 1976; Bilbao et al., 2000) assuming the weights of agents in the games are polynomial in the number of agents.

Given that computing the two power indices is already NP-hard, and only pseudo-polynomial or approximation algorithms are available to compute agents' power, it is reasonable that the manipulation algorithms we propose are also pseudo-polynomial since we necessarily need to use these power indices in computing agents' benefits during manipulation. (Aziz et al., 2011) have also shown that determining if there exists a beneficial merge is NP-hard using either the Shapley-Shubik or Banzhaf power indices. The same is true for determining the existence of beneficial annexation using the Banzhaf index.

4.2 Manipulation Algorithm for Merging

The brute force approach to determine a coalition that yields the most improved benefit in merging in a WVG is to simply enumerate all the possible coalitions of agents in the game and compute for each of these coalitions its benefit. We can then output the coalition with the highest value. Unfortunately, enumerating all the possible coalitions is exponential in the number of agents. Also, computing the power indices (to determine the factor of increment of each coalition) naively from their definitions means that we have two exponential time problems to solve. We provide an alternative approach.

Let procedure *PowerIndex*(G,i) be a pseudo-polynomial algorithm for computing the power index of an agent i in a WVG G of n agents for any of Shapley-Shubik and Banzhaf power indices according to (Matsui and Matsui, 2000). We first use *PowerIndex*(G,i) as a subroutine in the construction of a procedure, *GetMergeBenefit*(G,S). Procedure *GetMergeBenefit*(G,S) accepts a WVG G and a would-be manipulators' coalition, S. It first computes the sum of the individual power index of the assimilated agents in S using *PowerIndex*(G,i). Then, it alters G by replacing the sum of the weights of the assimilated agents in G with a single weight in a new game G' before computing the power of the bloc $\&S$ in G'. Finally, *GetMergeBenefit*(G,S) returns the factor of increment of the merged bloc $\&S$. Let $A(G)$ be the pseudo-polynomial running time of *PowerIndex*(G,i). Now, since $|S| \leq |I| = n$, procedure *GetMergeBenefit*(G,S) takes at most $O(n \cdot A(G))$ time which is pseudo-polynomial.

We now use *GetMergeBenefit*(G,S) to construct an algorithm that manipulators can use to determine a coalition that yields a good benefit in merging. We first argue that manipulators tend to prefer coalitions which are small in size because they are easier to form and manage. Also, intra-coalition coordination, communication, and other overheads increase with coalition size. Thus, we suggest a limit on the size of the manipulators' coalitions since it is unrealistic and impractical that *all* agents in a WVG will belong to the manipulators' coalition. This is also consistent with the assumptions of the previous work on annexation and merging (Aziz et al., 2011; Lasisi and Allan, 2011). We note, however, that limiting the manipulators' coalitions size this way does not change the complexity class of the problem as finding the coalition that yields the most improved benefit remains NP-hard even with such limitation.

Consider a WVG of n agents. Suppose the manipulators' coalitions have a limit, $k < n$, on the size of the members of the coalitions, i.e., the manipulators' coalitions, S, are bounded as $2 \leq |S| \leq k$. In this case, the number of coalitions that the manipulators need to examine is at most $O(n^k)$ which is polynomial in n. Specifically, the total number of these coalitions is:

$$\binom{n}{2} + \binom{n}{3} + \cdots + \binom{n}{k} = \sum_{j=2}^{k} \binom{n}{j}. \quad (3)$$

So, we have

$$\begin{aligned}
\sum_{j=2}^{k} \binom{n}{j} &= \sum_{j=2}^{k} \frac{n(n-1)\cdots(n-j+1)}{j!} \\
&\leq \sum_{j=2}^{k} \frac{n^j}{j!} \\
&\leq \sum_{j=2}^{k} \frac{n^j}{2^{j-1}} \\
&= \frac{n^2}{2^1} + \frac{n^3}{2^2} + \cdots + \frac{n^k}{2^{k-1}} = O(n^k).
\end{aligned}$$

Running *GetMergeBenefit*(G,S) while updating the most[1] improved benefit found so far from each of these coalitions requires a total running time of $O(n^k \cdot A(G))$ which is pseudo-polynomial time, and thus becomes reasonable to compute.

4.3 Manipulation Algorithm for Annexation

Our pseudo-polynomial manipulation algorithm for annexation provides a basic modification of the merge algorithm above. Specifically, we first replace the procedure *GetMergeBenefit*(G,S) with another procedure, *GetAnnexationBenefit*(G,i,S). The procedure *GetAnnexationBenefit*(G,i,S) accepts a WVG G, an annexer, i, and a coalition S to be assimilated by i. The procedure then returns the factor of increment or benefit of the assimilated bloc $\&(S \cup \{i\})$.

Again, we use *GetAnnexationBenefit*(G,i,S) to construct an algorithm that the annexer can use to determine the coalition that yields the most improved benefit in annexation. The method of construction of the algorithm is the same as that of the previous manipulation algorithm for merging with the exception that we add the weight of an annexer i to the weight of each coalition S and compare the power index $\Phi_{\&(S\cup\{i\})}(G')$ of the assimilated bloc in a new game G' to the power index $\Phi_i(G)$ of the annexer in the original game G. The annexer examines a polynomial number of coalitions of the agents assuming a limit $k < n$ on the size of each coalition. Since any of the n agents can be an annexer and the annexer will belong to any of the coalitions it annexes, the total number of coalitions examined by all the annexers is:

$$\begin{aligned}
&\binom{n}{1}\left[\binom{n-1}{1} + \binom{n-1}{2} + \cdots + \binom{n-1}{k-1}\right] \\
&\qquad = \binom{n}{1} \sum_{j=1}^{k-1} \binom{n-1}{j}. \qquad (4)
\end{aligned}$$

Bounding this equation using similar approach as in Equation 3 shows that Equation 4 is $O(n^k)$. Thus, as before, the manipulation algorithm for annexation also runs in pseudo-polynomial time, with a total running time of $O(n^k \cdot A(G))$.

5 INFORMED HEURISTIC SEARCH STRATEGIES

To further improve the performance of the manipulation algorithms, we use heuristics. In this section, we provide descriptions of search infrastrutures and enhancements to complement the performance of the manipulation algorithms.

5.1 Merging Heuristics

The search space for the manipulation algorithm for merging is the polynomial number of coalitions of size at most k (see Equation 3) as described earlier. It is important to point out that the computation of the power indices of the manipulators in the original game and the power index of the bloc formed by these agents in a new game account for most of the computational time required by this algorithm. We also note that it is unclear to the manipulators on how to determine a coalition that is beneficial without having to compute and compare the power indices of the manipulators in both games.

Since we seek to find the coalition with the most improved benefit among these coalitions, it is not difficult to see that the algorithm is prone to engaging in irrelevant computation of power indices of agents in the two games for coalitions whose merging are disadvantageous. We define evaluation criteria that we use to prune away such coalitions without having to compute the power indices of the blocs in the new games, thus gaining ample computational time. The basic idea of the evaluation criteria is to prune away all coalitions having their factor of increment less than

[1] We refer to the most improved benefit among the $O(n^k)$ polynomial coalitions and not from the original 2^n coalitions since we have restricted each manipulators' coalition size to a constant $k < n$.

or equal to the estimated minimum possible factor of increment in the altered WVGs. The criteria are defined for both Shapley-Shubik and Banzhaf indices.

First, consider merging using the Shapley-Shubik index. Let G be a WVG of n agents. Let the Shapley-Shubik power index of an agent i in a game G be $\varphi_i(G)$. Consider a manipulators' coalition $S \subseteq I$ with k agents. Let agents $i_1, i_2, \ldots, i_k$ be the distinguished k manipulators in S that want to merge into a single bloc $\&S$. Let Π_{n-k} be the set of all permutations of the remaining $n-k$ agents in G (i.e., not including the k manipulators). Consider a certain permutation $\pi \in \Pi_{n-k}$ in which we insert all the k manipulators starting from the r-th position in π (where r is any arbitrary position in π), and such that one of the manipulators is pivotal for π. There are $k!$ permutations in G for π in which the members of S occur together beginning at position r. For example, consider a permutation $\pi = 1,2,3$ of other agents in G which excludes the three manipulators i_1, i_2, and i_3. The 3! permutations in G for π when all the manipulators appear together and starting at the 3-rd position are: $\pi_1 = 1,2,i_1,i_2,i_3,3$, $\pi_2 = 1,2,i_1,i_3,i_2,3$, $\pi_3 =, \cdots, \pi_6 = 1,2,i_3,i_2,i_1,3$.

Now, consider a permutation $f(\pi)$ of agents in the altered game G' obtained from π by inserting the bloc $\&S$ (formed by agents $i_j \in S$) at the r-th position in $f(\pi)$. It is easy to see that the $k!$ permutations $\pi_1, \cdots, \pi_{k!}$ for π in G when the manipulators appear together from the r-th position correspond to exactly one permutation $f(\pi)$ in G'. Also, since one of the manipulators is pivotal for each of the permutations $\pi_1, \cdots, \pi_{k!}$ for π in G, $\&S$ is also pivotal for the corresponding $f(\pi)$ in G'. Observe that counting the number of distinct permutations $\pi \in \Pi_{n-k}$ in G in which we insert all the manipulators at certain positions and such that one of them is pivotal for each π provides a lower bound on the number of times the blocs formed by these agents in G' will be pivotal. That it is a lower bound is clear as the manipulators may also be pivotal in other cases when they do not all appear together.

In our implementation, we first count the number of times each of the manipulators is pivotal when they all follow one another in G. We then pick the smallest among these numbers denoted by $|\Pi_n^*|$. Now, if S merges to form a bloc $\&S$, then, the number of agents in the new game G' is $n-k+1$. We estimate the Shapley-Shubik power index of the bloc in G' as $\frac{|\Pi_n^*|}{(n-k+1)!}$. We compare the estimated power index of the bloc in G' to the sum of the Shapley-Shubik power indices, $\sum_{i\in S}\varphi_i(G)$, of the manipulators in G. Specifically, if $\frac{|\Pi_n^*|}{(n-k+1)!} \leq \sum_{i\in S}\varphi_i(G)$, we eliminate the manipulators' coalition S as the coalition cannot possibly be a candidate coalition that provides the most improved benefit to the manipulators.

Second, consider merging using Banzhaf index. Let $\eta_i(G)$ be the number of coalitions for which an agent i is critical in G. Also, denote by $\beta_i(G)$ the Banzhaf power index of agent i in G. As before, we consider a manipulators' coalition $S \subseteq I$ with k agents. Let agents $i_1, i_2, \ldots, i_k$ be the distinguished k manipulators in S that want to merge into a single bloc $\&S$. Let Γ_{n-k} be the set of all losing coalitions of the remaining $n-k$ agents in G (i.e., not including the k manipulators). Consider a certain coalition $C \in \Gamma_{n-k}$ in which the inclusion of at least one of agents $i_j \in S$ makes $C \cup \{i_j\}$ a winning coalition and such that at least one of agents i_j is critical for $C \cup \{i_j\}$. There are multiple such winning coalitions that can be formed from the union of C and the subsets of S depending on the quota of the game.

For example, let $G = [23,20,10,11,15;50]$ be a WVG of five agents $I = \{1,2,3,4,5\}$ in order. Let $S = \{3,4,5\}$ be a set of manipulators. Consider a losing coalition $C = \{1,2\}$ which excludes the three manipulators. There are three winning coalitions: $C_1^w = C \cup \{3\}$, $C_2^w = C \cup \{4\}$, and $C_3^w = C \cup \{5\}$ that can be formed from the union of C and the subsets of S such that at least one agent in S is critical in the resultant set. Note that adding two members of S to C would yield coalitions in which no agent is critical.

Now, consider a winning coalition $f(C)$ of agents in the altered game G' obtained from the union of the losing coalition C in game G and the bloc $\&S$ (formed by agents $i_j \in S$). It is easy to see that all the winning coalitions $C_1^w, \cdots, C_m^w$ (where $m \in \mathbb{N}$) obtained from C in G correspond to exactly one winning coalition $f(C)$ in G'. Also, since at least one of the manipulators is critical for each of the winning coalitions $C_1^w, \cdots, C_m^w$, the bloc $\&S$ is also critical for the corresponding winning coalition $f(C)$ in G'. Observe that counting the number of distinct losing coalitions $C \in \Gamma_{n-k}$ in which the inclusion of agents $i_j \in S$ makes $C \cup \{i_j\}$ a winning coalition and such that at least one of agents i_j is critical for $C \cup \{i_j\}$ gives the number of times the bloc formed by the manipulators in G' is critical.

In our implementation, we compute the sum $\sum_{i\in S}\eta_i(G)$ of the number of times all the manipulators are critical in G, and then compute the number $\eta_{\&S}(G')$ of times the bloc formed by the manipulators will be critical in G' as described above. In order to estimate the power of the bloc $\&S$ in G' we need to know the number of times for which each of the other agents in G' other than the bloc is also critical. These numbers are not available. Since we already know the number of times all agents in G are critical, we estimate the number of times for which each agent i (other than &S) in game G' would be critical as $\eta_i(G') = \frac{\eta_i(G)}{\tau}$, where τ is defined as a measure to scale down the number of times an agent is critical in G to G'. This is required since there are more agents in the original game G than G'. The number of coalitions for which the non manipulating agents in G is critical is always more than the number of coalitions for which they are critical in G'. More precisely, we estimate the scaling factor between the two games using the following ratio $\tau = \frac{\sum_{i \in S} \eta_i(G)}{\eta_{\&S}(G')}$. We now compute the estimated Banzhaf power index of the bloc in G' as $\beta_{\&S}(G') = \frac{\eta_{\&S}(G')}{\eta_{\&S}(G') + \sum_{i \in I \setminus S} \eta_i(G')}$. We compare the estimated power index of the bloc in G' to the sum of the Banzhaf power indices, $\sum_{i \in S} \beta_i(G)$, of the manipulators in G. Specifically, if $\beta_{\&S}(G') \leq \sum_{i \in S} \beta_i(G)$, we prune the manipulators' coalition S as the coalition cannot possibly be a candidate coalition that provides the most improved benefit to the manipulators.

5.2 Annexation Heuristic

We recall the definition of annexation in Section 2 and from (Machover and Felsenthal, 2002; Aziz et al., 2011), the power of the assimilated bloc in an altered WVG is compared to the power of the annexer in the original game. By this definition, intuition suggests that annexation should always be advantageous. This intuition is indeed true using the Shapley-Shubik index to compute agents' power. However, there exists situations where annexation is disadvantageous for the annexer using the Banzhaf index. See (Machover and Felsenthal, 2002; Aziz and Paterson, 2009; Aziz et al., 2011) for different examples of WVGs where annexation is disadvantageous for the annexer using the Banzhaf index. This case where annexation results in power decrease for the annexer is refer to as the *bloc paradox* (Machover and Felsenthal, 2002). Furthermore, (Aziz et al., 2011) have also shown that determining whether a player can benefit from annexing a given coalition is NP-hard for the Banzhaf index.

Recall again from Equation 4 that the annexer needs to examine only a polynomial number of assimilated coalitions of size at most $k-1$ to find the most improved power gain. It is also known that in computing agents' power index in a WVG using both the Shapley-Shubik and Banzhaf indices, the power index of an agent with a higher weight cannot be less than the power index of an agent with a smaller weight (Bachrach et al., 2010). In our case, since we are restricting the manipulators' coalition size to k, the assimilated coalitions with maximal weights are those of size $k-1$. Based on this observation and the fact due to the bloc paradox as discussed above, it is enough to check only the assimilated coalitions of size exactly $k-1$ in order for an annexer to find the coalition with the most improved benefit using the two power indices. There are only $\binom{n}{1}\binom{n-1}{k-1}$ such assimilated coalitions to be considered when the n agents act as an annexer in turn.

6 EXPERIMENTAL RESULTS

We have studied the performance of the two manipulation algorithms. As noted in the introduction, the simple random approach to manipulation via annexation and merging seems unintelligent. Thus, it is impractical that strategic agents would employ such method. We make a simple modification to this method which provides manipulators with higher average factor of increment. The modification involves the selection of the best factor of increment from three random choices (which we refer to as the *best-of-three* method). We compare the results of our search-based method with those of the simple random and best-of-three methods. However, for clarity of presentation, we show our results compared with only those of the best-of-three method.

We randomly generate WVGs. The weights of agents in each game are randomly chosen so that all weights are integers and drawn from a uniform distribution over the range $[1,W]$, where $W \in \{10,20,30,40,50\}$. We have chosen different weight distributions in order to provide some generalization of the performance of the two methods under different conditions. We run two different set of tests in which the number of agents, n, in each of the original WVGs is either 10 or 20 while the number of assimilated agents, k, is chosen to be either 5 or 10. When creating a new game, the quota, q, of the game is randomly generated such that $\frac{1}{2}w(I) < q \leq w(I)$, where $w(I)$ is the sum of the weights of all agents in the game.

Using the manipulation algorithms, the simple random and the best-of-three methods, the power index of an assimilated bloc formed by annexation in an altered game is compared to the power index of the annexer in the original game. Similarly, the power index of the assimilated bloc formed by merging is compared to the sum of the original power indices of the agents in the merged bloc. The factor of increment (decrement) by which the annexer (or the merged bloc) gains (loses) in the annexation (or merging) is computed. We repeat each experiment 100 times and compute the average factor of increment.

6.1 Results for Merging

Figure 1 shows the benefits from merging for both the

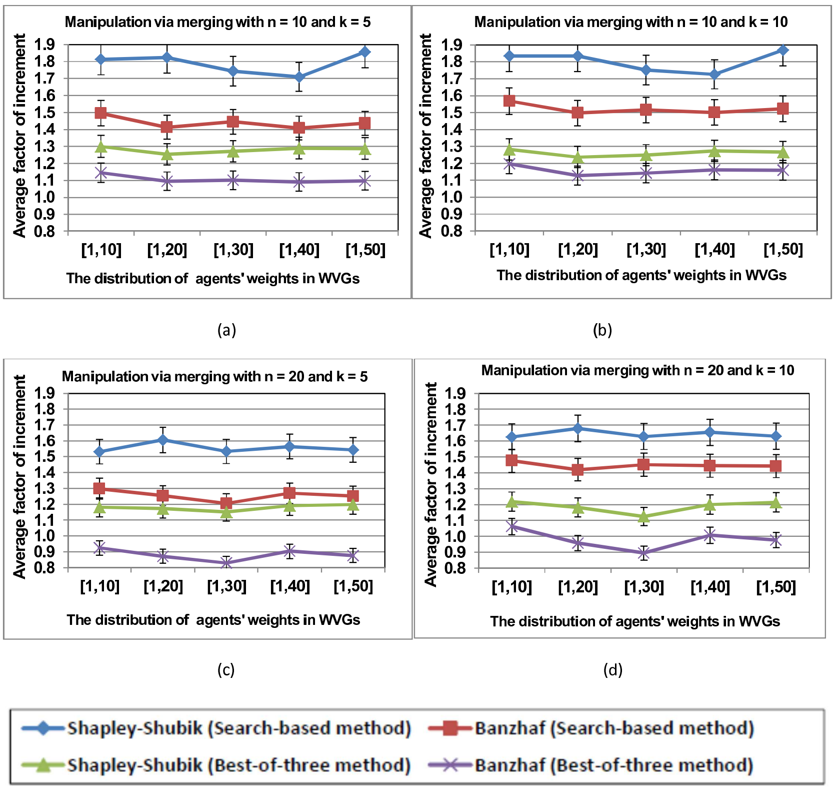

Figure 1: The average factor of increment for merging for the search-based and best-of-three methods using different agents' weights distributions. (a) $n = 10$ and $k = 5$ (b) $n = 10$ and $k = 10$ (c) $n = 20$ and $k = 5$ (d) $n = 20$ and $k = 10$.

best-of-three method and our search-based approach for various values of n, k, and W using the two power indices. The x-axis indicates the weight distributions of agents while the y-axis is the average factor of increment achieved by manipulating agents. The error bars in this and the subsequent figure indicate 5% error amounts in the average factor of increment.

We found from the data of Figures 1(a)-(d) that our search-based method achieves improvement in benefits of 28% to 45% more than those of the best-of-three method in merging for various values of n, k, and W, and for the two power indices. Since this percentage increment of the search-based approach over the best-of-three method can be achieved with only a polynomial amount of work, then, manipulators are more likely to seek a much improved power gain in merging using the search-based approach.

6.2 Results for Annexation

Figure 2 shows the average factor of increment from annexation for both the best-of-three method and the search-based approach for various values of n, k, and W using the two power indices.

It is clear from the figure that the average factor of increment found by the search-based approach for the two power indices are higher than those of the corresponding power indices using the best-of-three method. Specifically, we found from the data of Figures 2(a)-(d) that the search-based method achieves improvement in benefits of over 300% more than those of the best-of-three method in annexation for various values of n, k, and W, and for the two power indices. Again, this percentage increment of the search-based approach over the best-of-three method can be achieved with only a polynomial amount of work. Thus, we conclude that manipulation via merging and annexation is more serious than was presented in the simple random simulation of the previous work.

7 RELATED WORK

Weighted voting games and power indices are widely studied (Brams, 1975; Felsenthal and Machover, 1998; Laruelle, 1999). Prominent real-life situations where WVGs have found applications include the United Nations Security Council, the International Monetary Fund (Leech, 2002; Alonso-Meijide and Bowles, 2005), the Council of Ministers, and the Eu-

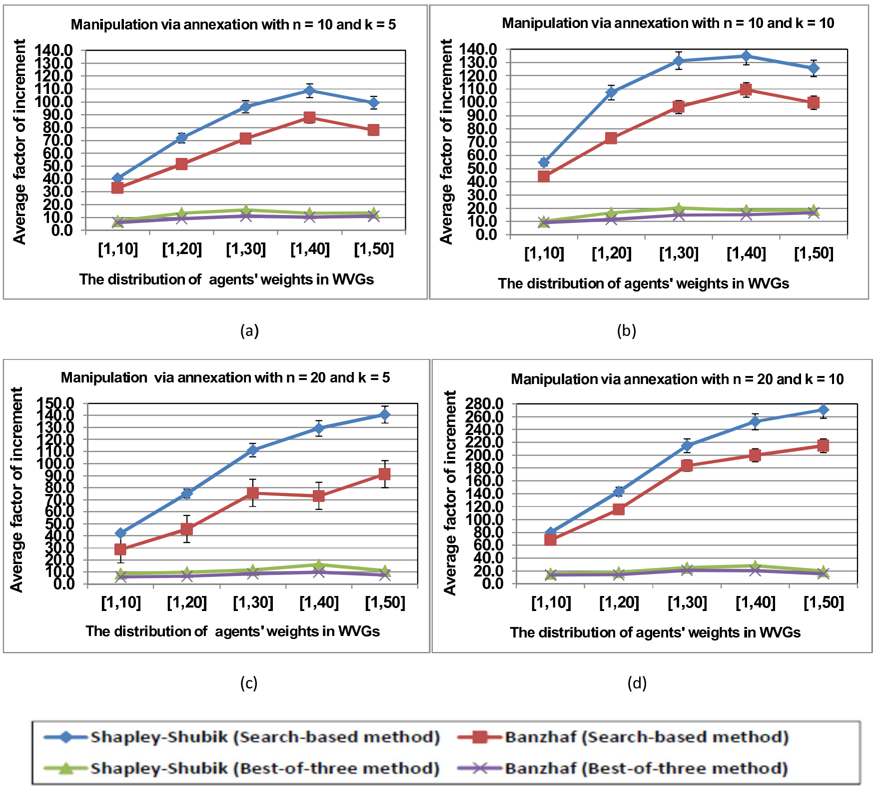

Figure 2: The average factor of increment for annexation for the search-based and best-of-three methods using different agents' weights distributions. (a) $n = 10$ and $k = 5$ (b) $n = 10$ and $k = 10$ (c) $n = 20$ and $k = 5$ (d) $n = 20$ and $k = 10$.

ropean Community (Felsenthal and Machover, 1998).

The need to compensate agents from jointly derived payoff in WVGs has also necessitated the assignment of power to players. A widely accepted method for measuring power of agents in WVGs uses power indices. *Fairness* in the assignment of power to players in a game is also a concern of most of the power indices. The two most prominent and widely used power indices are Shapley-Shubik (Shapley and Shubik, 1954) and Banzhaf (Banzhaf, 1965) power indices. Other power indices found in the literature include Deegan-Packel (Deegan and Packel, 1978), Johnsoton (Johnston, 1978), and Holler-Packel (Holler and Packel, 1983) power indices.

Computing the Shapley-Shubik and Banzhaf power indices of players in WVGs is NP-hard (Matsui and Matsui, 2001). The power indices of voters using any of Shapley-Shubik and Banzhaf power indices can be computed in pseudo-polynomial time using dynamic programming (Matsui and Matsui, 2000). Efficient exact algorithms using generating functions (Brams and Affuso, 1976; Bilbao et al., 2000) also exist for both the Shapley-Shubik and Banzhaf power indices for WVGs where the weights of all agents are restricted to integers. There are also approximation algorithms (Fatima et al., 2007; Bachrach et al., 2010) for computing the Shapley-Shubik and Banzhaf power indices in WVGs.

We now consider false-name manipulation via annexation and merging in WVGs. (Machover and Felsenthal, 2002) originally studied annexation and alliance (or merging) in WVGs. They consider when the blocs formed by annexation or merging are advantageous or disadvantageous. They show that using the Shapley-Shubik power index, it is always advantageous for a player to annex some other players in the game. However, this is not true for Banzhaf power index. Furthermore, they show that merging can be advantageous or disadvantageous for the two power indices. In contrast to our work, they do not consider the extent to which the agents involved in annexation or merging may gain, which we study in this paper.

(Aziz et al., 2011) have also considered the computational aspects of the problem of annexation and merging in WVGs. They show that determining if there exists a beneficial merge in a WVG is NP-hard using both Shapley-Shubik and Banzhaf indices. The same is also true for determining the existence of beneficial annexation using the Banzhaf index. Our work differ from that of these authors as we provide a comparison of the extent of power gain or benefits that are possible for manipulating agents in a restricted version of this problem using the two indices.

8 CONCLUSIONS

We extend the simple random simulation of (Lasisi and Allan, 2011) on susceptibility of power indices to annexation and merging in WVGs to consider a much improved benefit achievable by manipulating agents. Using similar assumptions for annexation and meging as employed in the simulation of (Lasisi and Allan, 2011), we show that manipulators need to do only a polynomial amount of work to find a much improved benefit and then present two search-based pseudo-polynomial manipulation algorithms that manipulators can use.

We provide a modified version of the simple random approach that considers the best benefit from three random choices (which we refer to as the *best-of-three* method) that we compare results of our search-based approach with. Experimental results show that our search-based method achieves improvement in benefits of over 300% more than those of the best-of-three approach in annexation, while the improvement in benefits is 28% to 45% more than the best-of-three method in merging for all the WVGs we considered. We conclude that since this percentage increment of the search-based approach over the best-of-three method for both annexation and merging can be achieved with only a polynomial amount of work, and using pseudo-polynomial algorithms, then, manipulators are more likely to seek for a much improved power gain when faced with annexation and merging in WVGs. Thus, we advance the state of the art by showing that annexation and merging can be more serious than as presented in the previous work.

ACKNOWLEDGEMENTS

This work is supported by NSF research grant #0812039 entitled “Coalition Formation with Agent Leadership”.

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