False Name Manipulations in Weighted Voting Games: Susceptibility of Power Indices

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Abstract. The splitting of weights into smaller sizes by agents in a weighted voting game and the distribution of the new weights among several false identities with the intent of payoff or power increase in a new game consisting of the original agents as well as the false identities is called *false name manipulation*. In this paper, we study false name manipulations in weighted voting games focusing on the power indices used in evaluating agents' payoff in such games. We evaluate the susceptibility to false name manipulations in weighted voting games of the following power indices, namely, Shapley-Shubik, Banzhaf and Deegan-Packel indices when an agent splits into several false identities. Our experimental results suggest that the three power indices are susceptible to false name manipulations when an agent splits into several false identities. However, the Deegan-Packel power index is more susceptible than Shapley-Shubik and Banzhaf indices.
General Terms: Algorithms, Economics, Theory

Key words: Agents, Manipulations, Power indices, Trust

1 Introduction

Cooperation among self-interested autonomous agents in multiagent environments is fundamental for agents to successfully achieve goals for which they lack enough resources and skills. The level of skills and amount of resources of agents varies, hence the need for agents' cooperation to complete tasks that are otherwise difficult for individual agents to achieve or for which better results (than working independently) can be attained. One way of modelling such cooperation is via *weighted voting games*.

Weighted voting games (WVGs) are mathematical abstractions of voting systems. In a voting system, voters express their opinions through their votes by electing candidates to represent them or influence the passage of bills. Each member voters, V, has an associated weight $w : V \rightarrow Q^+$. A voter's weight is the number of votes controlled by the voter, and this is the maximum number of votes she is permitted to cast. The *homogeneous voting system* is a special case in which all voters have unit weight [7]. In our context, a subset of agents, called the *coalition*, wins in a WVG, if the sum of the weights of the individual agents in the coalition meets or exceeds a certain threshold called the *quota*. In the case of the more traditional homogeneous voting system, the winning coalition (WC) is determined by the majority of the agents. On the other hand, in the usual WVGs with all agents having different weights, a coalition with sum of the individual agents' weights meeting or exceeding the quota determines the WC.

It is natural to naively think that the numerical weight of an agent directly determines the corresponding strength of the agent in a WVG. The measure of the strength of an agent is its *power*. This is the ability of an agent to influence the decision-making process. Consider, for example, a WVG of three voters, a_1, a_2, and a_3 with respective weights $6, 3$, and 1. Suppose the quota for the game is 10, then it is clear that a coalition consisting of all the three voters is needed to win the game. Thus, each of the voters a_1, a_2, and a_3 are of equal importance in achieving the WC. Hence, they each have equal power irrespective of their weight distribution in that every voter is necessary for a win.

A strategic agent may alter a game in anticipation of power increase by splitting its weight among several false agents that are not in the original game. Bachrach and Elkind [3] refer to this action as *false-name manipulation*. The new game consists of all the previous agents and the several false identities into which the manipulating agent splits. The power of the agent is thus the sum of the powers of all its false identities. This agent anticipates that the value of its accumulative power to be at least the value in the original game. Bachrach and Elkind [3] and Aziz and Paterson [1] show that this anticipation of power increase due to splitting into exactly two false identities is not achieved at all times. There are cases when the cumulative power of the false identities remains the same or even decreases compared with the original power of the agent.

Common measures of agents' power are the Shapley-Shubik, Banzhaf, and Deegan-Packel power indices [8]. Bachrach and Elkind [3] and Aziz and Paterson [1] evaluate the effects of false name manipulation when an agent splits into exactly two false identities using Shapley-Shubik and Banzhaf indices respectively.

To date, there has been practically no work on the effect of false name manipulations when an agent splits into more than two identities and thus, remains unexplored [1]. In this paper, we evaluate the susceptibility to false name manipulations in WVGs of the following power indices, namely, Shapley-Shubik, Banzhaf, and Deegan-Packel indices for the case when an agent splits into several false identities. The more resistant to manipulation a power index is the better. Hence, agents' motivation towards manipulation is thus reduced. This provides some assurance of identity, which is crucial for establishing and maintaining trustworthy interactions. The remainder of the paper is organized as follows. Section 2 discusses related work. Section 3 provides the definitions and notations used in the paper. In Sect. 4, we provide examples to illustrate false name manipulation when an agent splits into exactly two false identities using the Deegan-Packel index. In Sect. 5, we provide experimental evaluation of susceptibility of the three power indices to false name manipulations. We conclude in Sect. 6 and provide directions for future work.

2 Related Work

WVGs and power indices are widely studied [1],[3],[4],[8]. WVGs have many applications, including economics, political science, neuroscience, threshold logic, reliability theory, distributed systems [2], and multiagent systems [3]. Aziz et al. provide a brief discussion of some applications of WVGs. Prominent instances of weighted voting problems are in the United Nations Security Council, the Electoral College of the United States and the International Monetary Fund [2].

The study of WVGs has also necessitated the need to fairly determine the strength or power of players in a game. This is because the power of a player in a game provides information about the relative importance or criticality of that player in the game compared to other players. To evaluate the power of the players, power indices such as Shapley-Shubik, Banzhaf, and Deegan-Packel indices are commonly employed [8]. These power indices satisfy the axioms that characterize a power index [3], have gained wide usage in political arena, and are the main power indices found in the literature [6]. Computing the power indices of players using any of Shapley-Shubik, Banzhaf index, or Deegan-Packel index is NP-hard [8], and the problem is also #P-complete for Shapley-Shubik and Banzhaf. However, these values can be computed in pseudo-polynomial time by dynamic programming [5], [8]. There are also approximation algorithms for computing the power indices using Shapley-Shubik and Banzhaf indices [4].

These power indices have been defined on the framework of subsets of WCs in the game they seek to evaluate. A wide variation in the results they provide can be observed. This is due partly to the different definitions and methods of computation of the associated subsets of the WCs. Then, comes the question of which of the power indices is the most resistant to manipulation in a WVG. The choice of a power index depends on a number of factors, namely, the a priori properties of the index, the axioms characterizing the power indices, and the context of decision making process under consideration [6].

False name manipulation has been studied in the context of non-cooperative games [3] and in open anonymous environments, such as the internet [9]. False name manipulation is hard to discover and can be effective in such environments. The menace can take different forms, such as agents providing multiple identities, two or more agents merging identities to form a single agent, non disclosure of full status (in the form of hiding skills) by agents or even a combination of these forms [9]. The maiden study of this behavior in the context of WVG is the work of Bachrach and Elkind [3]. This action involves an agent splitting into a number of false agents with the intent that the cumulative power index value of the false agents exceeds the original value. They use the Shapley-Shubik index to evaluate agent power and consider the case when agents splits into exactly two false agents. The extent to which agents increase or decrease their Shapley power are also bounded. Similar results using the Banzhaf power index were obtained by Aziz and Paterson [1].

3 Definitions and Notations

We give the following definitions and notations used throughout the paper.

Weighted Voting Game: Let I $= \{1, \cdots, n\}$ be a set of agents. Let $\mathbf{w} = \{w_1, \cdots, w_n\}$ be the corresponding positive integer weights of the agents in order. Let S be a non empty set of agents. $S \subseteq I$ is a coalition. A WVG G with quota q involving agents I is defined as $G = [w_1, \cdots, w_n; q]$. Denote by $w(S)$, the weight of a coalition S derived from the summation of the individual weights of agents in S i.e. $w(S) = \sum_{i \in S} w_i$. A coalition, S, wins in the game G if $w(S) \geq q$ otherwise it loses. So that simultaneously there can be a single WC, q is constrained as follows $\frac{1}{2}w(I) < q \leq w(I)$.

Simple Voting Game: Each of the $2^{|I|}$ coalitions $S \subseteq I$ has an associated function $v : S \to \{0, 1\}$. The value 1 implies a win for the coalition and 0 a loss. In the game $G, v(S) = 1$ if $w(S) \geq q$ and 0 otherwise.

Dummy and Critical Agents: An agent $i \in S$ is *dummy* if its weight in S is not needed for S to be a WC, i.e. $w(S - \{i\}) \geq q$. Otherwise it is *critical* to coalition S, i.e. $w(S) \geq q$ and $w(S - \{i\}) < q$.

Unanimity Weighted Voting Game: A WVG in which there is a single WC and every agent is critical to the coalition is a *unanimity weighted voting game.*

Shapley-Shubik Power Index: The Shapley-Shubik power index is one of the oldest power indices and has been used widely to analyze political power. The index quantifies the marginal contribution of an agent to the grand coalition. Each agent in a permutation is given credit for the win if the agents preceding it do not form a WC but by adding the agent in question, a WC is formed. The power index is dependent on the number of permutations for which an agent is critical. For the $n!$ permutations of agents used in determining the Shapley-Shubik index, there exists exactly one critical agent in each of the permutations. Adopting Bachrach and Elkind's notation [3], we denote by Π the set of all permutations of n agents in a WVG G. Let $\pi \in \Pi$ define a one-to-one mapping where $\pi(i)$ is the position of the ith agent in the permutation order. Denote by $S_\pi(i)$, the predecessors of agent i in π, i.e., $S_\pi(i) = \{j : \pi(j) < \pi(i)\}$. The Shapley-Shubik value of the ith agent in G is given by

$$\varphi_i(G) = \frac{1}{n!} \sum_{\pi \in \Pi} [v(S_\pi(i) \cup \{i\}) - v(S_\pi(i))] \tag{1}$$

Banzhaf Power Index: Another index that has also gained wide usage in the political arena is the Banzhaf power index. Unlike the Shapley-Shubik index, its computation depends on the number of WCs in which an agent is critical. There can be more than one critical agent in a particular WC. The Banzhaf index, $\beta_i(G)$, of agent i in the same game, G, as above is given by

$$\beta_i(G) = \frac{\eta_i(G)}{\sum_{i \in I} \eta_i(G)} \tag{2}$$

where $\eta_i(G)$ is the number of coalitions in which i is critical in G.

Deegan-Packel Power Index: The Deegan-Packel power index is also found in the literature for computing power indices. The computation of this power index for an agent i takes into account both the number of all the minimal winning coalitions (MWCs) in the game as well as the sizes of the MWCs having i as a member [8]. Thus, it is more impressive to be one in three (who elicited the win) rather than one in ten. A WC $C \subseteq I$ is a MWC if every proper subset of C is a losing coalition, i.e. $w(C) \geq q$ and $\forall T \subset C, w(T) < q$. The Deegan-Packel power index, $\gamma_i(G)$, of an agent i in G is given by

$$\gamma_i(G) = \frac{1}{|MWC|} \sum_{S \in MWC_i} \frac{1}{|S|} \tag{3}$$

where MWC$_i$ are the sets of all MWCs in G that include i.

Susceptibility of Power Index to Manipulation: Let Φ be a power index. Denote by $\Phi_i(G)$, the power of an agent i in a WVG G. Suppose i alters G by splitting into k false identities having weights $w_{i1}, \cdots, w_{ik}$. Let G' be the resulting game such that $\sum_{j=1}^{k} w_{ij} = w_i$ and the weights of all other agents in G' remain constant. We say that Φ is susceptible to manipulation if there exists $\sum_{j=1}^{k} \Phi_{ij}(G') > \Phi_i(G)$, and the split is *advantageous.* If $\sum_{j=1}^{k} \Phi_{ij}(G') < \Phi_i(G)$, then the split is *disadvantageous* while it is *neutral* when $\sum_{j=1}^{k} \Phi_{ij}(G') = \Phi_i(G)$.

4 False Name Manipulations with Deegan-Packel Index

In this section, we provide examples to illustrate false name manipulation in WVGs using the Deegan-Packel power index to compute power. We consider the case where an agent splits into exactly two false identities in the new game. Splitting into more than two false identities is considered in the next section.

Example 1. Splitting Advantageous

Let $G = [5, 4, 3; 7]$ be a WVG of three agents $1, 2$, and 3 having respective weights $5, 4$, and 3 with quota $q = 7$. The game has three MWCs $\{\{1, 2\}, \{1, 3\}, \{2, 3\}\}$. Consider agent 1. The Deegan-Packel index of this agent computed using (3) above is $\gamma_1(G) = \frac{1}{3}$. Suppose the agent splits into two new false agents 1_a and 1_b with respective weights 3 and 2. We have a new game $G' = [3, 2, 4, 3; 7]$. The MWCs for this game consist of $\{\{1_a, 2\}, \{1_a, 1_b, 3\}, \{2, 3\}\}$. The respective Deegan-Packel indices for agents 1_a and 1_b are $\gamma_{1_a}(G') = \frac{5}{18}$ and $\gamma_{1_b}(G') = \frac{1}{9}$. Clearly, the sum of the values of the two indices, namely $\gamma_{1_a}(G') + \gamma_{1_b}(G') = \frac{7}{18} > \gamma_1(G)$. Thus, the agent benefits from the split action by an increase in payoff.

Example 2. Splitting Disadvantageous

Let $G = [20, 10, 8, 8, 3, 2, 1, 1; 28]$ be a WVG. The Deegan-Packel index of agent 1 is $\gamma_1(G) = 0.2500$. Suppose this agent splits into 1_a and 1_b with weights 14 and 6 respectively in a new game $G' = [14, 6, 10, 8, 8, 3, 2, 1, 1; 28]$. The Deegan-Packel indices of agents 1_a and 1_b are 0.1339 and 0.1113 respectively. Hence, $\gamma_{1_a}(G') + \gamma_{1_b}(G') = 0.2452 < \gamma_1(G)$.

Example 3. Splitting Neutral

Let $G = [5, 4, 3; 7]$ be the same WVG as example 1. Suppose the agent splits into 1_a and 1_b with respective weights 4 and 1 in a new game $G' = [4, 1, 4, 3; 7]$. The Deegan-Packel indices of agents 1_a and 1_b are $\frac{1}{3}$ and 0 respectively. Hence, $\gamma_{1_a}(G') + \gamma_{1_b}(G') = \gamma_1(G)$, and the agent neither benefited nor incured a decrease in payoff.

5 Susceptibility of Power Indices to Manipulations

In this section, we demonstrate the susceptibility to false name manipulations in WVGs of the following power indices, namely, Shapley-Shubik, Banzhaf and Deegan-Packel power indices. We consider the more general case of agents splitting into more than two identities. For the sake of simplicity in our discussion, we provide the following assumptions that by no means invalidate the basic requirements of WVGs:

1. Only one of the agents is engaging in the manipulation at a time. We note that other agents also have similar motivation to split their weights in anticipation of power increase. In our future work, we intend to consider scenarios where multiple agents in WVGs simultaneously engage in false name manipulation.
2. All splits of agents' weights are into integer values. Otherwise we will have an exponential number of possible new weights to evaluate.
3. The weights of the false identities that an agent splits into are strictly greater than zero. By the null player axiom [6], assigning a weight of zero to an agent does not make the agent critical in all WCs.

5.1 Unanimity Weighted Voting Games

We recall that a WVG in which there is a single WC and such that every agent is critical to the coalition is a unanimity WVG game. Since all the agents in the WC of unanimity WVGs are critical, the total weights of all agents, $w(I)$ and the quota, q in such games satisfy the inequality, $w(I) \geq q$.

Proposition 1. *In a unanimity WVG with $q = w(I)$, if Banzhaf indices are used as payoffs of agents in a WVG, then it is beneficial for an agent to split up into several agents. The same holds for Shapley-Shubik power index [1].*

Proposition 2. *In a unanimity WVG with $q = w(I)$, if the Deegan-Packel index is used to compute power of agents, then it is advantageous for an agent to split up into several false agents.*

Proof. Let G be a unanimity WVG of n agents with quota $q = w(I)$. It is easy to see that the Deegan-Packel power index of every agent i in G, $\gamma_i(G) = \frac{1}{n}$. Suppose agent 1 splits into $m + 1$ false agents, then we have a new unanimity game, G' of $n + m$ agents. The Deegan-Packel power index of every agent i in G',$\gamma_i(G') = \frac{1}{n+m}$. Hence, the new Deegan-Packel power index of agent 1 is $\gamma_1(G') = \frac{m+1}{n+m} > \frac{1}{n}$ for $n > 1$. $\square$

The following theorem is immediate from propositions 1 and 2.

Theorem 1. *Let G be a unanimity WVG of n players with quota $q = w(I)$. Suppose an agent i splits into $k \geq 3$ false agents. The Shapley-Shubik, Banzhaf, and Deegan-Packel power index of agent i increases as its split size, k, increases.*

Corollary 1. *Let G be a unanimity WVG with $w(I) > q$. Let an agent i split into several false agents in a new game G'. Suppose the new game G' is also a unanimity WVG, then the splitting is advantageous for i if any of Shapley-Shubik, Banzhaf, and Deegan-Packel power index is use to compute the agent's power.*

5.2 General Weighted Voting Games

False name manipulation in the general case of the WVGs is more interesting as it provides more complex and realistic scenarios that are not well-understood. Of importance is that the number of WCs and MWCs changes in contrast to being static as observed with unanimity WVGs with $q = w(I)$. Thus, as the structure of the WVGs changes, so does the composition of the WCs and MWCs.

As mentioned in the introduction, the more detailed analysis on the effect of false name manipulations when an agent splits into more than two false agents remain unexplored [1]. The only closely related research are the NP-hardness results of finding a beneficial split in WVGs when a manipulating agent splits into at least two false agents. The hardness results are for the Shapley-Shubik [3] and Banzhaf [1] power indices. To the best of our knowledge, ours is the first paper to confirm the existence of beneficial splits when agents split into more than two false identities for the three power indices we consider.

We perform experiments to simulate the effect of manipulations by agents using the three power indices. The weights of our agents are chosen so that no weight is larger than ten. These weights are reflective of realistic voting procedures as the weights of agents in real voting are not too large [3] and as such are representative of WVGs. In the experiments, we randomly generate WVGs and assume only the first agent in the game is engaging in the manipulation, then determine the three power index values of this agent in the game. After this, we consider splits into at least two false identities by this agent while the weights of all other agents remain the same in the new games. Suppose the initial weight of the manipulating agent in the original game is n, we allow the agent, to split its weight among the false agents in the new games as follows, $\{\{n-1, 1\}, \{n-2, 1, 1\}, \cdots, \{1, 1, \cdots, 1\}\}$. The values of the power indices of the several false agents into which the manipulating agent splits are then added and compared with the power index of the agent in the original game. We generate 20,000 original WVGs for the experiments and allowed the manipulating agent to split its weight in each of the games. The numbers of the new games generated by the action of the manipulating agent depends on the initial weight of the agent in the original games.

Our experiments suggest the existence of beneficial splits when agents engage in such manipulations for the three power indices. However, the extent to which agents gain varies with the indices. The effect of this action is well noticed with the Deegan-Packel index as we found cases where agents improve their power index by more than four times the original value. On the other hand, the maximum gain attained while using any of Shapley-Shubik and Banzhaf index is less than a factor of two. The result suggests that Deegan-Packel power index is more susceptible to false name manipulations than the Shapley-Shubik and the Banzhaf indices. Hence, this may provides some motivation for agents to engage in manipulation in WVGs when the game is being evaluated with the Deegan-Packel index. We illustrate three games from the experiments in which an agent attains high factor splitting into more than two false agents for the three power indices.

Example 4. Consider the WVG $G = [6, 2, 2, 3, 10; 12]$. The gains of the manipulating agent (with weight 6) is depicted in Table 1 for the Shapley-Shubik power index.

Table 1. The splitting agent weight, Shapley-Shubik indices, and the factor of increment in the game $G = [6, 2, 2, 3, 10; 12]$.

Splitting Agent Weights	Shapley-Shubik Index	Factor Increment
$\{6\}$	0.1000	-
$\{5, 1\}$	0.1000	-
$\{4, 1, 1\}$	0.1238	1.2
$\{3, 1, 1, 1\}$	0.1429	1.4
$\{2, 1, 1, 1, 1\}$	0.1548	1.5
$\{1, 1, 1, 1, 1, 1\}$	0.1667	1.7

Example 5. Consider the WVG $G = [7, 8, 4, 8, 4; 16]$. The gains of the manipulating agent (with weight 7) is depicted in Table 2 for the Banzhaf power index.

Table 2. The splitting agent weight, Banzhaf indices, and the factor of increment in the game $G = [7, 8, 4, 8, 4; 16]$.

Splitting Agent Weights	Banzhaf Index	Factor Increment
$\{7\}$	0.1429	-
$\{6, 1\}$	0.1429	-
$\{5, 1, 1\}$	0.1429	-
$\{4, 1, 1, 1\}$	0.1429	-
$\{3, 1, 1, 1, 1\}$	0.1864	1.3
$\{2, 1, 1, 1, 1, 1\}$	0.2381	1.7
$\{1, 1, 1, 1, 1, 1, 1\}$	0.2672	1.9

Example 6. Consider the WVG $G = [8, 4, 9, 1, 4; 14]$. The gains of the manipulating agent (with weight 8) is depicted in Table 3 for the Deegan-Packel power index.

Table 3. The splitting agent weight, Deegan-Packel indices, the factor of increment, and the number of MWCs in the game $G = [8, 4, 9, 1, 4; 14]$.

Splitting Agent Weights	Deegan-Packel Index	Factor Increment	# of MWC
$\{8\}$	0.1667	-	5
$\{7, 1\}$	0.2143	1.3	7
$\{6, 1, 1\}$	0.2407	1.4	9
$\{5, 1, 1, 1\}$	0.3036	1.8	14
$\{4, 1, 1, 1, 1\}$	0.4207	2.5	29
$\{3, 1, 1, 1, 1, 1\}$	0.5344	3.2	57
$\{2, 1, 1, 1, 1, 1, 1\}$	0.6201	3.7	115
$\{1, 1, 1, 1, 1, 1, 1, 1\}$	0.6754	4.1	229

For this example, the number of MWCs in the original game is 5. They are $\{3, 4, 5\}, \{2, 3, 5\}, \{2, 3, 4\}, \{1, 3\}$, and $\{1, 2, 5\}$. Although the weight of agent 1, 8 is relatively high compared to some other agents in the game, it belongs to only two of the MWCs having two and three members. Of particular interest are agents 2 and 5 with weight 4 each, these agents belong to three MWCs with each of the coalitions having three members, hence, by (3), their power from the original game is greater than that of agent 1 with higher weight. Agent 1 thus has motivation to split its weight. So, as the agent splits its weight, it becomes active in more MWCs which improves its power index.

5.3 Simulation Results

We present the results of our extensive set of simulations. For our study, we generate $20,000$ original WVGs and allow manipulation of all the games by the manipulating agent. When starting a new game, all agents are randomly assigned weights in the current game and the quota of the game is also generated based on the weights assumed by the agents. We designate the first agent to be the manipulating agent. We have five agents in each of the original games. The maximum weight that can be asssumed by the manipulating agent is eight while all other agents can be up to ten. The least possible weights for any agent is one. Obviously, when the manipulating agent assumes a weight of one in a game, then it is not possible for it to split in such game. We keep the weight of the manipulating agent to be lower than other agents to limit the number of cases. Since all weigths are randomly generated, we have a handfull of different types of games that are representative of the WVGs. For example, we have many cases where the original weight of the manipulating agent is lower, higher or even the same as the weights of many or all agents in the original games.

We first consider how susceptibility to manipulation among the power indices compares when an agent is allowed to manipulate a game. This is achieved by comparing the population of the factor of increment attained by agents in different games for each indices with the split sizes (the number of false agents the original agent splits). We show a summary of the ease of manipulation by agents for the three indices in 20, 000 WVGs in Fig. 1. The x-axis indicates the split size while the y-axis is the average factor of increment achieved by agents in the 20, 000 WVGs for different split sizes. The high susceptibility of the Deegan-Packel index to manipulation can be observed from the figure. While the average factor of increment for manipulation rapidly grows with split size for this index, the growth for the Shapley-Shubik and Banzhaf does not appear to correlate with split sizes, and on average does not improve utility. From our experiments, many of the games are advantageous with the Deegan-Packel index while many are disadvantageous for both Shapley-Shubik and Banzhaf indices. This result is indicative of the ease by which each of the power indices is manipulable.

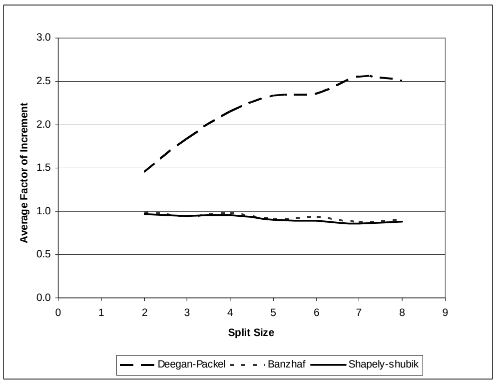

Fig. 1. Ease of Manipulation among the three Power Indices

The number of games that are advantageous, neutral, and disadvantageous for the three indices are also analyzed. Figure 2 shows that a larger number of the games are advantageous for Deegan-Packel index than for Shapley-Shubik and Banzhaf indices. Consider when 1, 000 games are generated, in more than 800 of the games are splitting advantageous for Deegan-Packel while we have less than 300 advantageous games for Shapley-Shubik and Banzhaf indices. Similarly, Fig. 3 shows the number of neutral games. We have more neutral games for Shapley-Shubik and Banzhaf than Deegan-Packel. Virtually none of the games are neutral for Deegan-Packel while about 90 of the games are neutral for Shapley-Shubik and Banzhaf indices out of a collection of 1, 000 games. Finally, Fig. 4 shows that there are fewer disadvantageous games for Deegan-Packel compare with Shapley-Shubik and Banzhaf indices. Clearly, the Deegan-Packel index is more susceptible to false name manipulations than Shapley-Shubik and Banzhaf indices.

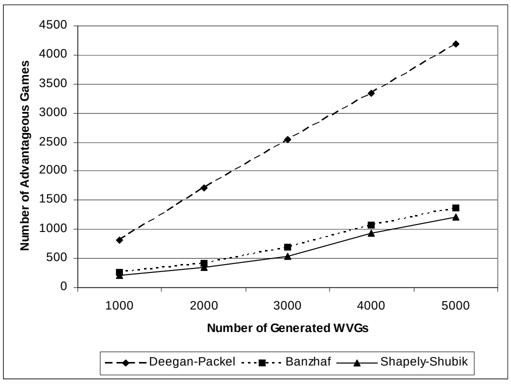

Fig. 2. Number of Advantageous Games among the Generated WVGs

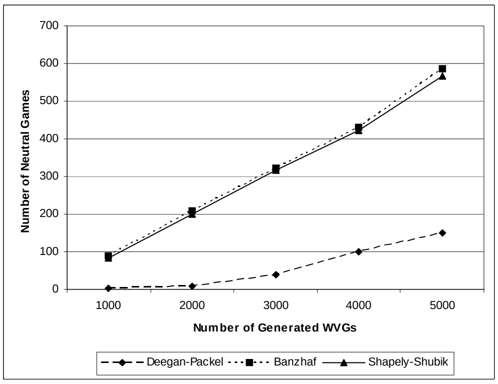

Fig. 3. Number of Neutral Games among the Generated WVGs

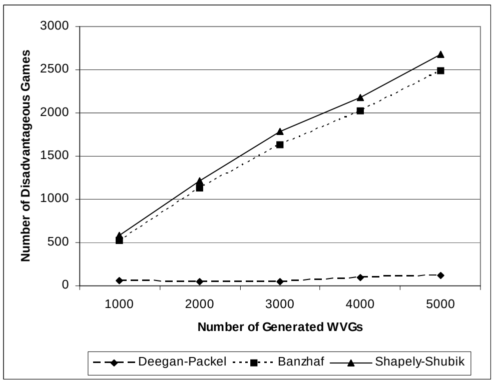

Fig. 4. Number of Disadvantageous Games among the Generated WVGs

6 Conclusions

In this paper, we evaluate the susceptibility to false name manipulations of the following power indices, namely, Shapley-Shubik, Banzhaf, and Deegan-Packel indices when an agent splits into several false identities in weighted voting games. We illustrate the susceptibility of the three indices through simulations of a large number of weighted voting games with a manipulating agent in each of the games. Our experimental results suggest that the three indices are susceptible to false name manipulations when an agent splits into several false identities. However, the Deegan-Packel index is more susceptible than Shapley-Shubik and Banzhaf indices, with Shapley-Shubik being the least susceptible. Hence, using Shapley-Shubik index to evaluate weighted voting games reduces agents' motivation towards false name manipulations. This provides some assurance of identity, which is crucial for establishing and maintaining trustworthy interactions.

Since our experimental results have suggested ideas on the extent to which each of the three indices are susceptible to false name manipulations, an obvious direction for future work is to provide theoretical bounds on the extent to which each of the indices are susceptible to false name manipulations when an agent splits into several false identities. It will also be interesting to come up with desirable properties that power indices should satisfy in order to prevent false name manipulations or prove that such properties are not achievable.

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