California 55; Texas 38; Florida 29; New York 29; Illinois 20; Pennsylvania 20; Ohio 18; Georgia 16; Michigan 16; North Carolina 15; New Jersey 14; Virginia 13; Washington 12; Arizona 11; Indiana 11; Massachusetts 11; Tennessee 11; Maryland 10; Minnesota 10; Missouri 10; Wisconsin 10; Alabama 9; Colorado 9; South Carolina 9; Kentucky 8; Louisiana 8; Connecticut 7; Oklahoma 7; Oregon 7; Arkansas 6; Iowa 6; Kansas 6; Mississippi 6; Nevada 6; Utah 6; Nebraska 5; New Mexico 5; West Virginia 5; Hawaii 4; Idaho 4; Maine 4; New Hampshire 4; Rhode Island 4; Alaska 3; Delaware 3; D.C. 3; Montana 3; North Dakota 3; South Dakota 3; Vermont 3; Wyoming 3

Total votes = 538 and quota = \((538 / 2) + 1\) = 270
Weighted Voting in the Electoral College

Choosing a president with the electoral college - whichever candidate achieves a weight of 270 wins
How Important is Each State?

- Where should candidates do most campaigning or spend campaign funds?
• What is the impact/strength of each state in a winning coalition?
The impact of a player/agent on the final decision is termed its **POWER**.
A Prominent Index for Measuring Power or Payoff

• Shapley-Shubik (1954)

(Φ)
So, why do we care about weighted voting systems?
Weighted Voting in Automated Decision-Making

- Weighted voting
- Threshold logic
- Distributed systems
- Network flow
- Search & rescue
- Multi-robot team formation
- Target detection
- Pattern recognition
- Safety monitoring
1. Computing *Single Large Party’s* Power

2. Bounds for Manipulation by *Merging*
The Shapley Value is Attractive

- Unique solution
- Fair solution

Computing the Shapley value in WVGs is #P-complete (Deng and Papadimitriou, 1994)
The Shapley Value is Attractive

- Unique solution
- Fair solution

Computing the Shapley value in WVGs

Is \textit{\#P-complete} (Deng and Papadimitriou, 1994)
[\{q; w_l, w_s, \ldots, w_s\}, \text{where } w_l > w_s \text{ and } w_s \geq 1\] 

\[m \text{ times}\] 

**Required**

- \(w_l < q\), otherwise, the large player can win in a game without forming coalitions with any of the small players.

- \(m \cdot w_s < q\), so that the small players also need the large player to win in a game.
Known Results until Now

- \( \varphi_l = \frac{w_l}{m+1}, \text{ for } w_s = 1 \)
- \( \varphi_l = \left\lceil \frac{w_l}{w_s} \right\rceil \frac{1}{m+1}, \text{ for } w_s > 1 \)

These results are incorrect!
Known Results until Now

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- $\varphi_l = \left\lceil \frac{w_l}{w_s} \right\rceil \frac{m}{m+1}$, for $w_s > 1$

These results are incorrect!
Proposed (Correct) Shapley Value Formula

$$\phi_l = \frac{m + 1 - \left\lceil \frac{q - w_l}{w_s} \right\rceil}{m + 1}$$

for $w_s \geq 1$
Manipulation by Merging (i.e., dishonest behavior)

Strategic agents misrepresenting their identities

- strategic agents
- false agent
Consider Electronic Negotiation

- Agents, $A = \{a_1, a_2, \ldots, a_n\}$, negotiating on how to allocate budget $B$
- A payoff method allocates, say, $P = \{p_1, p_2, \ldots, p_n\}$, to agents, $A$, respectively, based on their weights
- Suppose some strategic agents, $S \subset A$, merge their weights to form a single bloc, they may be able to increase their share of the budget

Here are the questions we seek to answer:

- What is the amount of damage that is caused to the non-manipulating agents?
- Analogously, what is the extent of budgets, payoffs, or power that manipulators may gain depending on the context under consideration?
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The Merging Problem - Using Shapley-Shubik Index

Assuming the bill requires a quota, \( q \in [111, 120] \)

<table>
<thead>
<tr>
<th>Parliament</th>
<th>Allocation</th>
</tr>
</thead>
<tbody>
<tr>
<td>A, 20</td>
<td>A, $3.33</td>
</tr>
<tr>
<td>B, 30</td>
<td>B, $20.00</td>
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<tr>
<td>C, 40</td>
<td>C, $20.00</td>
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<tr>
<td>D, 50</td>
<td>D, $28.33</td>
</tr>
<tr>
<td>E, 50</td>
<td>E, $28.33</td>
</tr>
</tbody>
</table>

$100 million spending bill
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Assuming the bill requires a quota, \( q \in [111, 120] \)

![Parliament Allocation Pie Chart]

<table>
<thead>
<tr>
<th>Parliament</th>
<th>Allocation</th>
</tr>
</thead>
<tbody>
<tr>
<td>ABD, 100</td>
<td>ABD, $66.67</td>
</tr>
<tr>
<td>C, 40</td>
<td>C, $16.67</td>
</tr>
<tr>
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$100 million spending bill
Finding optimal beneficial merge is **NP-hard** for Shapley-Shubik index (Aziz et. al. 2011)
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- NP-hardness is only a worst case measure, thus, agents may be satisfied with sub-optimal beneficial merge
Good News from Previous Work?

Finding optimal beneficial merge is \textbf{NP-hard} for Shapley-Shubik index (Aziz et. al. 2011)

- NP-hardness is \textit{only a worst case measure}, thus, agents may be satisfied with sub-optimal beneficial merge

- Real instances of WVGs are small enough that \textit{exponential amount of work} may not deter manipulators
Upper and Lower “bounding” the effects of manipulation by merging
We employ theoretical proofs with ideas from combinatorics and algorithmic game theory.
Until now, no result exists on the bounds when two or more strategic players merge into a bloc.
Contributions

We provide the first two non-trivial bounds for this problem using the Shapley-Shubik index. The two bounds are also shown to be asymptotically tight.
Results

Theorem 1: Upper Bound

Let \( G = [q; w_1, \ldots, w_n] \) be a WVG of \( n \) agents. If two manipulators, \( m_1 \) and \( m_2 \), merge their weights to form a bloc, \&S, in an altered game \( G' \), then, the Shapley-Shubik power, \( \varphi_{&S}(G') \), of the bloc in the new game, \( \varphi_{&S}(G') \leq \frac{n}{2}(\varphi_{m_1}(G) + \varphi_{m_2}(G)) \). Moreover, this bound is asymptotically tight.

Theorem 2: Lower Bound

Let \( G = [q; w_1, \ldots, w_n] \) be a WVG of \( n \) agents. If two manipulators, \( m_1 \) and \( m_2 \), merge their weights to form a bloc, \&S, in an altered game \( G' \), then, the Shapley-Shubik power, \( \varphi_{&S}(G') \), of the bloc in the new game, \( \varphi_{&S}(G') \geq \frac{n}{2(n-1)}(\varphi_{m_1}(G) + \varphi_{m_2}(G)) \). Moreover, this bound is asymptotically tight.
### Open Problems

<table>
<thead>
<tr>
<th>Merging</th>
<th>Lower Bound</th>
<th>Upper Bound</th>
</tr>
</thead>
<tbody>
<tr>
<td>$k = 2$</td>
<td>This paper</td>
<td>This paper</td>
</tr>
<tr>
<td>$k &gt; 2$</td>
<td>?</td>
<td>?</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Splitting</th>
<th>Lower Bound</th>
<th>Upper Bound</th>
</tr>
</thead>
<tbody>
<tr>
<td>$k = 2$</td>
<td>Bachrach &amp; Elkind ’08</td>
<td>Bachrach &amp; Elkind ’08</td>
</tr>
<tr>
<td>$k &gt; 2$</td>
<td>Lasisi &amp; Allan ’14</td>
<td>Lasisi &amp; Allan ’14</td>
</tr>
</tbody>
</table>

**Table:** Summary of bounds for manipulations in WVGs
Table 1: Bounds for merging when the number of strategic agents, $k = 2$ (i.e., $m_1$ and $m_2$) or $k > 2$. $n$ is the number of agents in the initial game $G$, and $G'$ is the resulting game after manipulation.

<table>
<thead>
<tr>
<th>Bounds</th>
<th>Shapley-Shubik index</th>
<th>Banzhaf index</th>
</tr>
</thead>
<tbody>
<tr>
<td>Upper ($k = 2$)</td>
<td>$\varphi_{SS}(G') \leq \frac{n}{2} (\varphi_{m_1}(G) + \varphi_{m_2}(G'))^*$</td>
<td>$?$</td>
</tr>
<tr>
<td>Lower ($k = 2$)</td>
<td>$\varphi_{SS}(G') \geq \frac{n}{2(n-1)} (\varphi_{m_1}(G) + \varphi_{m_2}(G'))^*$</td>
<td>$?$</td>
</tr>
<tr>
<td>Upper ($k &gt; 2$)</td>
<td>$?$</td>
<td>$?$</td>
</tr>
<tr>
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<td>$?$</td>
<td>$?$</td>
</tr>
</tbody>
</table>

* (Lasisi & Lasisi, 2015)
Wrap up

So, why do we care about these BOUNDS?