The Shapley Value in Voting Games:

Computing Single Large Party's Power and Bounds for Manipulation by Merging

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by

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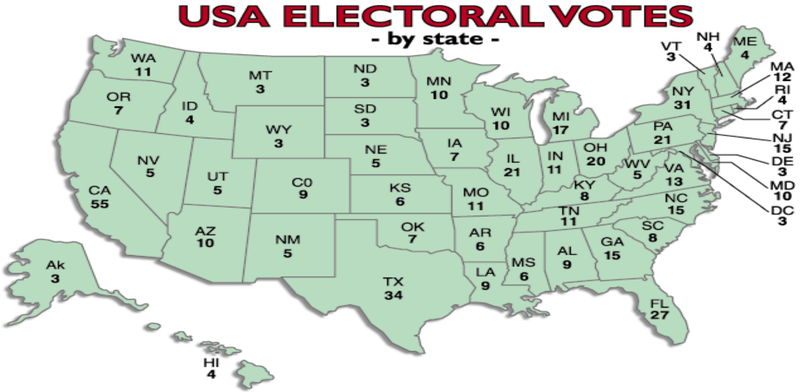
Distribution of Electoral Votes in the United States

California 55; Texas 38; Florida 29; New York 29; Illinois 20; Pennsylvania 20; Ohio 18; Georgia 16; Michigan 16; North Carolina 15; New Jersey 14; Virginia 13; Washington 12; Arizona 11; Indiana 11; Massachusetts 11; Tennessee 11; Maryland 10; Minnesota 10; Missouri 10; Wisconsin 10; Alabama 9; Colorado 9; South Carolina 9; Kentucky 8; Louisiana 8; Connecticut 7; Oklahoma 7; Oregon 7; Arkansas 6; Iowa 6; Kansas 6; Mississippi 6; Nevada 6; Utah 6; Nebraska 5; New Mexico 5; West Virginia 5; Hawaii 4; Idaho 4; Maine 4; New Hampshire 4; Rhode Island 4; Alaska 3; Delaware 3; D.C. 3; Montana 3; North Dakota 3; South Dakota 3; Vermont 3; Wyoming 3

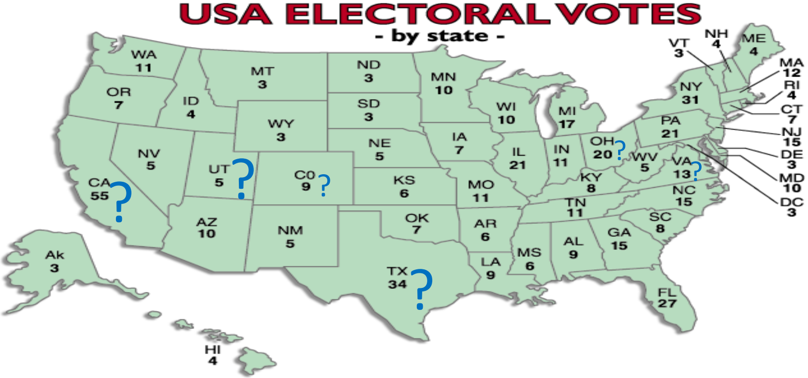
Total votes = 538 and quota = (538 / 2) + 1 = 270

Weighted Voting in the Electoral College



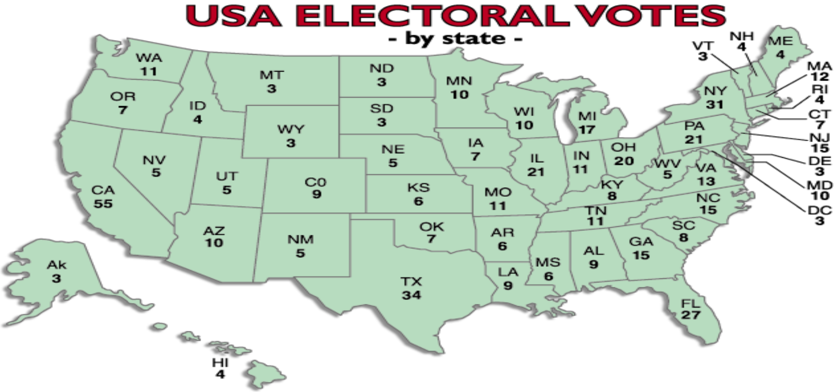
- Choosing a president with the electoral college - whichever candidate achieves a weight of 270 wins

How Important is Each State?



- Where should candidates do most campaigning or spend campaign funds?

Analogously



- What is the impact/strength of each state in a winning coalition?

Power

The impact of a player/agent on the final decision is termed its POWER.

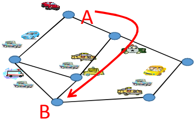
A Prominent Index for Measuring Power or Payoff

- Shapley-Shubik (1954)

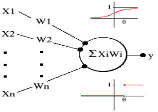
(φ)

So, why do we care about weighted voting systems?

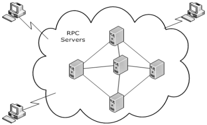
Weighted Voting in Automated Decision-Making



- network flow

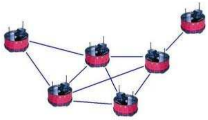


- threshold logic

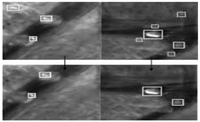


- distributed systems

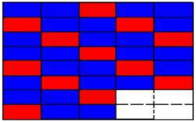
- search & rescue



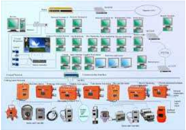
- Multi-robot team formation



- Target detection



- Pattern recognition



- Safety monitoring

1. Computing Single Large Party's Power
2. Bounds for Manipulation by Merging

The Shapley Value is Attractive

- Unique solution
- Fair solution

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Computing the Shapley value in WVGs

Is #P-complete (Deng and Papadimitriou, 1994)

Single Large Party's Voting Game

$$[q; w_l, \underbrace{w_s, \ldots, w_s}_{m \text{ times}}], \text{ where } w_l > w_s \text{ and } w_s \geq 1$$

Required

- $w_l < q$, otherwise, the large player can win in a game without forming coalitions with any of the small players
- $m \cdot w_s < q$, so that the small players also need the large player to win in a game.

Known Results until Now

- $\varphi_l = \frac{w_l}{m+1}$, for $w_s = 1$
- $\varphi_l = \frac{\lceil w_l / w_s \rceil}{m+1}$, for $w_s > 1$

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These results are incorrect!

Proposed (Correct) Shapley Value Formula

$$\varphi_l = \frac{m + 1 - \lceil \frac{q - w_l}{w_s} \rceil}{m + 1}$$

$$\text{for } w_s \geq 1$$

Manipulation by Merging (i.e., dishonest behavior)

Strategic agents misrepresenting their identities

- strategic agents

- false agent

Consider Electronic Negotiation

- Agents, $A = \{a_1, a_2, \ldots, a_n\}$, negotiating on how to allocate budget B
- A payoff method allocates, say, $P = \{p_1, p_2, \ldots, p_n\}$, to agents, A, respectively, based on their weights
- Suppose some strategic agents, $S \subset A$, merge their weights to form a single bloc, they may be able to increase their share of the budget

Motivation / Problem

Consider Electronic Negotiation

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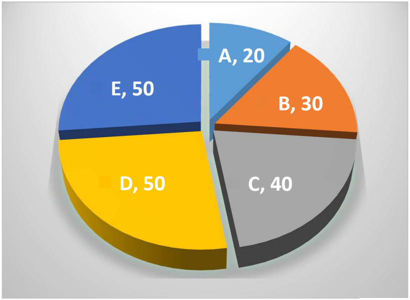
Here are the questions we seek to answer

- What is the amount of damage that is caused to the non manipulating agents?
- Analogously, what is the extent of budgets, payoffs, or power that manipulators may gain depending on the context under consideration?

The Merging Problem - Using Shapley-Shubik Index

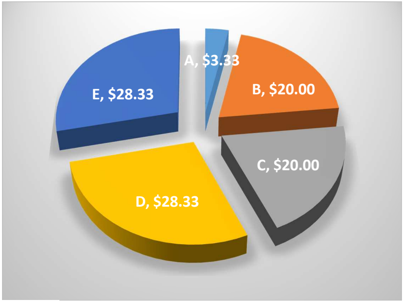
Assuming the bill requires a quota, $q \in [111, 120]$

Parliament



# of representatives in political parties

Allocation



Amount allocated to political parties (in millions)

$100 million spending bill

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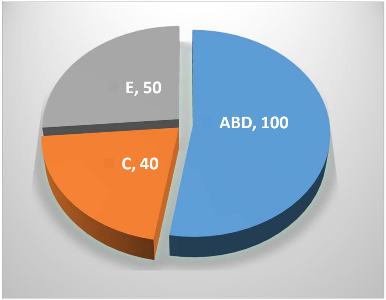

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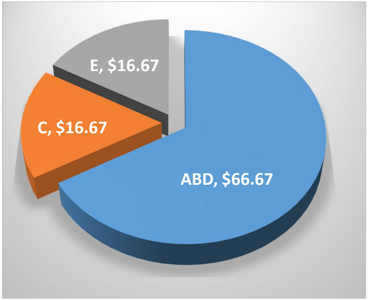

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Good News from Previous Work?

Finding optimal beneficial merge is **NP-hard** for Shapley-Shubik index (Aziz et. al. 2011)

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- Real instances of WVGs are small enough that exponential amount of work may not deter manipulators

Methodology - Bounds

- Upper and Lower "bounding" the effects of manipulation by merging

Approach

We employ theoretical proofs with ideas from combinatorics and algorithmic game theory.

Results

Until now, no result exists on the bounds when two or more strategic players merge into a bloc

Contributions

We provide the first two non-trivial bounds for this problem using the Shapley-Shubik index. The two bounds are also shown to be asymptotically tight.

Results

Theorem 1: Upper Bound

Let $G = [q; w_1, \ldots, w_n]$ be a WVG of n agents. If two manipulators, m_1 and m_2, merge their weights to form a bloc, $\&S$, in an altered game G', then, the Shapley-Shubik power, $\varphi_{\&S}(G')$, of the bloc in the new game, $\varphi_{\&S}(G') \leq \frac{n}{2}(\varphi_{m_1}(G) + \varphi_{m_2}(G))$. Moreover, this bound is asymptotically tight.

Theorem 2: Lower Bound

Let $G = [q; w_1, \ldots, w_n]$ be a WVG of n agents. If two manipulators, m_1 and m_2, merge their weights to form a bloc, $\&S$, in an altered game G', then, the Shapley-Shubik power, $\varphi_{\&S}(G')$, of the bloc in the new game, $\varphi_{\&S}(G') \geq \frac{n}{2(n-1)}(\varphi_{m_1}(G) + \varphi_{m_2}(G))$. Moreover, this bound is asymptotically tight.

Open Problems

Merging	Lower Bound	Upper Bound
$k = 2$	This paper	This paper
$k > 2$	?	?

Splitting	Lower Bound	Upper Bound
$k = 2$	Bachrach & Elkind '08	Bachrach & Elkind '08
$k > 2$	Lasisi & Allan '14	Lasisi & Allan '14

Table: Summary of bounds for manipulations in WVGs

Future Work

Table 1: Bounds for merging when the number of strategic agents, $k = 2(i.e., m_1$ and $m_2)$ or $k > 2$. n is the number of agents in the initial game G, and G' is the resulting game after manipulation

Bounds	Shapley-Shubik index	Banzhaf index
Upper $(k = 2)$	$\varphi_{\&S}(G') \leq \frac{n}{2}(\varphi_{m_1}(G) + \varphi_{m_2}(G))*$	?
Lower $(k = 2)$	$\varphi_{\&S}(G') \geq \frac{n}{2(n-1)}(\varphi_{m_1}(G) + \varphi_{m_2}(G))*$	?
Upper $(k > 2)$	?	?
Lower $(k > 2)$	?	?

* (Lasisi & Lasisi, 2015)

Wrap up

So, why do we care about these BOUNDS?